* $f(x) = a^x$ AND $f(x) = \log_a x$ are inverses

WS #4-4
Logarithmic Functions ; A logarithm is a name for a certain exponent.

1. You will be responsible to read the section completely and review the definitions and application of the following:

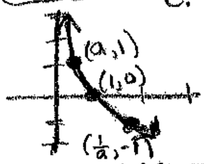
A. Logarithmic function to the base a where $a > 0$, and $a \neq 1$, is denoted by $y = \log_a x$ (read as "y is the logarithm to the base a of x") and is defined by $y = \log_a x$ if and only if $x = a^y$

$y = \log_a x$ (log form)
$a^y = x$ (exponential form)

B. Domain of Logarithmic Functions

- The domain of $y = \log_a x$ is $x > 0$
- Range of log funct = Domain of exp. funct $(-\infty, \infty)$
- Domain of log funct = Range of exponential function $(0, \infty)$

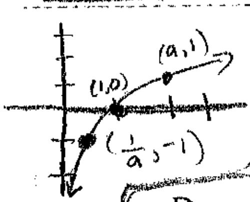
$0 < a < 1$



C. Properties of Logarithmic Functions $F(x) = \log_a x$

1. $D: (0, \infty)$; $R: (-\infty, \infty)$
2. x-int $(1,0)$; no y-intercept
3. Vertical Asymptote → the y-axis $(x = 0)$
4. Function is decreasing if $0 < a < 1$ + inc. if $a > 1$
5. The graph is smooth + continuous; contains the points $(1,0), (a,1), (\frac{1}{a}, -1)$
6. Graphs

$a > 1$



D. Natural Logarithmic Functions : Log function w/base 'e'

$y = \ln x$ is the same as $y = \log_e x$

$y = \ln x$ if $x = e^y$ • $y = \ln x$ + $y = e^x$ are inverses

E. Common Logarithmic Functions

$y = \log x \Leftrightarrow x = 10^y$
(log form) (exp form)

- log function w/ base 10
- $y = \log x$ + $y = 10^x$ are inverses
- If base of log function is not indicated, it is understood as 10.

F. Logarithmic Equations

- Equations that contain logs
- must check each apparent solutions in the original equ. + discard any extraneous solutions.
- In $\log_a M$, both a + M are positive and $a \neq 1$

2. Change to logarithmic form:

A. $1.2^3 = m$

$\log_{1.2} m = 3$

B. $e^b = 9$

$\log_e 9 = b$

C. $a^4 = 24$

$\log_a 24 = 4$

3. Change to exponential form:

A. $\log_a 4 = 5$

$a^5 = 4$

B. $\log_b e = -3$

$b^{-3} = e$

C. $\log_3 5 = c$

$3^c = 5$

4. Evaluate

A. $\log_2 16 \rightarrow 2^y = 16$

$\boxed{y = 4}$

B. $\log_3 \frac{1}{27} \rightarrow 3^y = \frac{1}{27}$

$3^y = 3^{-3}$

$\boxed{y = -3}$

8C. $\frac{100}{6} = \frac{6e^{12.77x}}{6}$

$\frac{50}{3} = e^{12.77x}$

$\ln\left(\frac{50}{3}\right) = 12.77x$ → $x = 0.22$

If 0.22 concentration of alc in blood, then 100% risk of an accident

• Solve the inequality for B

−	+	−
(−2)	(0)	(2)

(boundaries at −1 and 1)

• Since $|x| > 0$, as long as $x \neq 0$ the domain for C is:

5. Find the domain of;

A. $F(x) = \log_2(x+3)$

Domain: $x+3 > 0 \rightarrow x > -3$ or $(-3, \infty)$

B. $G(x) = \log_5\left(\frac{1+x}{1-x}\right)$

$\frac{1+x}{1-x} > 0$

D: $-1 < x < 1$ or $(-1, 1)$

C. $h(x) = \log_{\frac{1}{2}}|x|$

$(-\infty, 0)$ or $(0, \infty)$

6. Give the transformations for:

A. $f(x) = \ln x$ to $g(x) = -\ln(x+2)$ → reflect across x-axis; shift left 2 units.

B. $f(x) = \log x$ to $g(x) = 3\log(x-1)$ → vertical stretch by a factor of 3; right 1 unit.

A. Mult each y-coord by (−1) and add (−2) to each x-coordinate

B. Mult each y-coord by (3) and add (1) to each x-coordinate

7. Solving logarithmic equations;

A. $\log_3(4x-7) = 2$

① rewrite in exponential form + solve

$3^2 = 4x - 7$

$9 = 4x - 7$

$\frac{16}{4} = \frac{4x}{4}$ → $\boxed{x = 4}$

$\log_3(4 \cdot 4 - 7) = 2$

$\log_3 9 = 2$

→ $3^2 = 9$ ✓

B. $\log_x 64 = 2$

① rewrite in exp. form + solve

$x^2 = 64$

$x = \pm 8$

② check

$\log_8 64 = 2$

$8^2 = 64$ ✓

$\boxed{x = 8}$

* we can discard $x = -8$ b/c the base of a logarithm must always be positive.

• $\log_a M$ → both a + M must be positive

C. $e^{2x} = 5$

① Change the exponential to logarithmic + solve (take ln of each side).

$\ln e^{2x} = \ln 5$ * ln + e are inverses of each other so they cancel.

$\frac{\cancel{2}x}{\cancel{2}} = \frac{\ln 5}{2}$

$\boxed{x = \frac{\ln 5}{2} \approx 0.805}$

8D. $\frac{20}{6} = \frac{6e^{12.77x}}{6}$

$\frac{10}{3} = e^{12.77x}$

$\frac{\ln\left(\frac{10}{3}\right)}{12.77} = \frac{12.77x}{12.77}$

$.094 = x$

• If a driver has a blood alc of 9.4% or higher, they should be arrested + charged w/a DUI

8. The concentration of alcohol in a person's blood is measurable. Recent medical research suggests that the risk R (given as a percent) of having an accident while driving a car can be modeled by the equation $R = 6e^{kx}$ where x is the variable concentration of alcohol in the blood and k is a constant.

A. Suppose that a concentration of alcohol in the blood of 0.04 results in a 10% risk (R=10) of an accident. Find the constant k in the equation. Graph $R = 6e^{kx}$ using the k value.

B. Using this value of k, what is the risk if the concentration is 0.17?

C. Using the same value of k, what concentration of alcohol corresponds to a risk of 100%?

D. If the law asserts that anyone with a risk of having an accident of 20% or more should not have driving privileges, at what concentration of alcohol in the blood should a driver be arrested and charges with a DUI?

A. $\frac{10}{6} = \frac{6e^{.04K}}{6}$ → $\frac{5}{3} = e^{.04K}$ → $\frac{\ln\frac{5}{3}}{.04} = \frac{.04K}{.04}$ → $\boxed{K = 12.77}$

B. $R = 6e^{.17(12.77)} = \underline{52.6}$ → If there is a concentration of alcohol in the blood of 0.17, the risk of an accident is about $\boxed{52.6\%}$